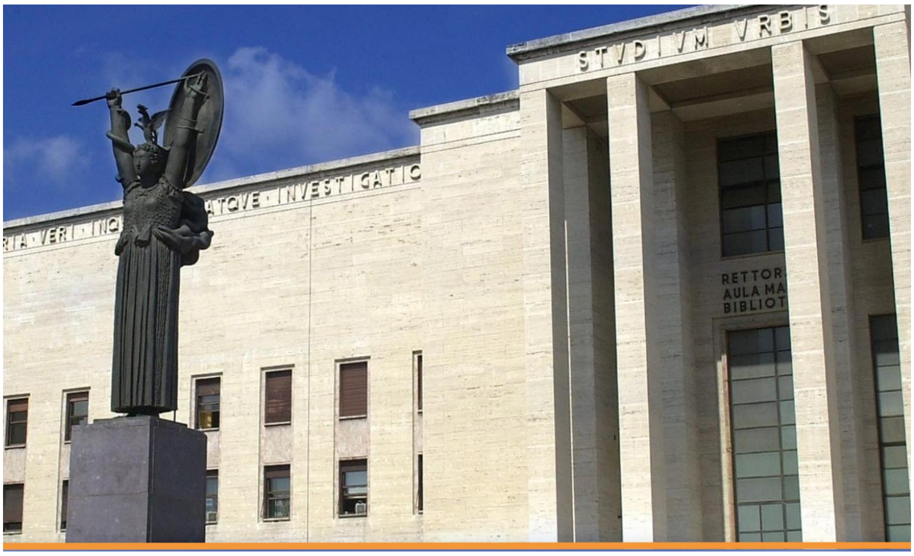
Mathematical Challenges in Quantum Mechanics 2018

February 19 – 24, 2018, Roma (Italy)
“Sapienza” University of Rome

PROGRAMME



Monday 19/02/2016

08:30 - 09:00 Opening & Registration

09:00 - 09:50 **Rupert L. Frank** – The ionization problem I

10:00 - 10:30 Coffee Break

10:30 - 11:20 **Rupert L. Frank** – The ionization problem II

11:30 - 12:20 **Stefan Teufel** – Adiabatic theorems in quantum mechanics I

12:30 - 14:30 Lunch Break

14:30 - 17:50 Contributed Talks

18:00 - 18:50 **Martin Zirnbauer** – Bott periodicity and the “periodic table” of topological insulators and superconductors

18:50 - Welcome Cocktail

Contributed Talks

Session 1 (room III)

14:30 - 14:50 **William Borrelli** – Effective Dirac equations in honeycomb structures

15:00 - 15:20 **Biagio Cassano** – Self-adjointness for the Dirac operator with Coulomb-type potentials

15:30 - 15:50 **Andrew Comech** – Stability of solitary waves in nonlinear Dirac equation with concentrated nonlinearity with broken $SU(1,1)$ symmetry

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Matteo Gallone** – Self-adjoint realisations of the Dirac-Coulomb Operator

17:00 - 17:20 **Sergey Morozov** – Dirac operators with Coulomb potentials

17:30 - 17:50 **Stefano Pasquali** – Dynamics of the nonlinear Klein-Gordon equation in the nonrelativistic limit

Session 2 (room Picone)

14:30 - 14:50 **Lea Boßmann** – Derivation of the 1d Gross-Pitaevskii equation from the 3d quantum many-body dynamics of strongly confined bosons

15:00 - 15:20 **Christian Brennecke** – Gross-Pitaevskii Dynamics for Bose-Einstein Condensates

15:30 - 15:50 **Viktor Gerasimenko** – On evolution equations of processes of creation and propagation of correlations in quantum systems

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Michael Hott** – Derivation of the Hartree equation for compound Bose gases in the mean field limit

17:00 - 17:20 **Jinyeop Lee** – Rate of Convergence towards Hartree Dynamics with Singular Interaction Potential

17:30 - 17:50 **Simone Rademacher** – From Hartree dynamics to the relativistic Vlasov equation

Tuesday 20/02/2016

09:00 - 09:50 **Rupert L. Frank** – The ionization problem III

10:00 - 10:30 Coffee Break

10:30 - 11:20 **Rupert L. Frank** – The ionization problem IV

11:30 - 12:20 **Nilanjana Datta** – Concentration of quantum states from quantum functional and transportation cost inequalities

12:30 - 14:30 Lunch Break

14:30 - 17:50 Contributed Talks

18:00 - 18:50 **Alessandro Giuliani** – Universality of the Hall conductivity in interacting electron systems

Contributed Talks

Session 1 (room III)

14:30 - 14:50 **Giovanni Antinucci** – Universal edge transport in interacting Hall systems

15:00 - 15:20 **Fabio Deelan Cunden** – Free fermions and the classical compact groups

15:30 - 15:50 **Giuseppe De Nittis** – Linear Response Theory: An Analytic-Algebraic Approach

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Giuseppe Florio** – Feynman graphs and the large dimensional limit of multipartite entanglement

17:00 - 17:20 **Domenico Monaco** – Kubo formula for the quantum (spin) Hall conductivity: a microscopic derivation

17:30 - 17:50 **Clément Tauber** – Conductivity and conductance: analysis of Kubo-like spin transport in topological insulators

Session 2 (room Picone)

14:30 - 14:50 **Giulia Basti** – Universal low-energy behavior in a quantum Lorentz gas with Gross-Pitaevskii potentials

15:00 - 15:20 **Thomas Norman Dam** – Non-existence of ground states in the translation invariant Nelson model

15:30 - 15:50 **Orif Ibrogimov** – Spectral analysis of the spin-boson Hamiltonian with two photons for arbitrary coupling

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Hans Konrad Knörr** – The adiabatic behaviour of a bound state diving into continuum

17:00 - 17:20 **Vitaly Moroz** – Groundstates and radial solutions to Schrödinger-Poisson-Slater equations at the critical frequency

17:30 - 17:50 **Marco Olivieri** – Quasi-Classical Limit for the Pauli-Fierz Model

Wednesday 21/02/2016

09:00 - 09:50 **Rupert L. Frank** – The ionization problem V

10:00 - 10:30 Coffee Break

10:30 - 11:20 **Rupert L. Frank** – The ionization problem VI

11:30 - 12:20 **Benjamin Schlein** – Bogoliubov theory for excitation spectra of interacting Bose gases I

12:30 - 13:20 **Sergio Albeverio** – Quantum fields and point interactions

13:20 - 15:00 Lunch Break

15:00 - 16:30 Round Table with Young Participants

Round Table (room E)

- **Gianfausto Dell'Antonio** (chair)

- **Francesco Ferrulli**
- **Mario Laux**
- **Matthiew Liew**
- **Iuliia Meshkova**
- **Bernard M. Pfirsch**
- **Nicolai Rothe**
- **Oliver Siebert**

Thursday 22/02/2016

09:00 - 09:50 **Stefan Teufel** – Adiabatic theorems in quantum mechanics II

10:00 - 10:30 Coffee Break

10:30 - 11:20 **Stefan Teufel** – Adiabatic theorems in quantum mechanics III

11:30 - 12:20 **Benjamin Schlein** – Bogoliubov theory for excitation spectra of interacting Bose gases II

12:30 - 14:30 Lunch Break

14:30 - 17:50 Contributed Talks

18:00 - 18:50 **Gregory Berkolaiko** – Local nodal surplus and nodal count distribution of graphs with disjoint loops

Contributed Talks

Session 1 (room III)

14:30 - 14:50 **Simon Becker** – Hamiltonian of contact interaction of Ter-Martyrosyan-Skornyakov (TMS) type

15:00 - 15:20 **Marko Erceg** – Friedrichs operators as dual pairs and contact interactions

15:30 - 15:50 **Davide Fermi** – Some results on scattering theory for delta interactions concentrated on deformed planes

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Marilena Ligabò** – Quantum boundary conditions

17:00 - 17:20 **Thomas Moser** – Stability of a fermionic $N+1$ particle system with point interactions

17:30 - 17:50 **Silvestro Fassari** – On the spectrum of the two-dimensional Schrödinger Hamiltonian with the isotropic harmonic confinement or the isotropic pyramidal one in the presence of a central point perturbation

Session 2 (room Picone)

14:30 - 14:50 **Andreas Deuchert** – Bose-Einstein Condensation in a Dilute, Trapped Gas at Positive Temperature

15:00 - 15:20 **Dinh Thi Nguyen** – Many-body blow-up profile of ground states of boson stars

15:30 - 15:50 **Alessandro Olgiati** – Ground state properties of mixtures of condensates

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Daniele Dimonte** – Phase transitions for a rotating Bose-Einstein condensate: the third critical speed

17:00 - 17:20 **Matthias Geyer** – An adiabatic approach to constrained quantum systems with spin-orbit interaction

17:30 - 17:50 **Davide Lonigro** – A generalization of the Friedrichs-Lee Hamiltonian and its singular coupling limit

Friday 23/02/2016

09:00 - 09:50 **Benjamin Schlein** – Bogoliubov theory for excitation spectra of interacting Bose gases III

10:00 - 10:30 Coffee Break

10:30 - 11:20 **Benjamin Schlein** – Bogoliubov theory for excitation spectra of interacting Bose gases IV

11:30 - 12:20 **Stefan Teufel** – Adiabatic theorems in quantum mechanics IV

12:30 - 14:30 Lunch Break

14:30 - 18:20 Contributed Talks

18:30 - 19:20 **Marcel Griesemer** – On the 2d Fermi-polaron

Contributed Talks

Session 1 (room III)

14:30 - 14:50 **Per Moosavi** – Universal relations in non-equilibrium CFT

15:00 - 15:20 **Ian Jauslin** – Nematic liquid crystal phase in a system of interacting dimers

15:30 - 15:50 **Alessandro Duca** – Global exact controllability of the bilinear Schrödinger potential type models on compact quantum graphs

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Francesco Calogero** – On the quantization of peculiar Hamiltonians

17:00 - 17:20 **Alexander Stottmeister** – Weyl quantization and lattice-gauge theory

17:30 - 17:50 **Lorenzo Zanelli** – An homogenization approach for the inverse spectral problem of periodic Schrödinger operators

18:00 - 18:20 **Annalisa Panati** – Control of fuctuations and heavy tails for heat variation in the two-time measurement framework

Session 2 (room Picone)

14:30 - 14:50 **Juan Pablo Borgna** – Optical solitons in nematic liquid crystals: model with saturation effects

15:00 - 15:20 **Li Chen** – On the Bogoliubov-de Gennes Equations

15:30 - 15:50 **Emanuela L. Giacomelli** – Surface Superconductivity in Presence of Corners

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Cyrill Muratov** – A Non-local Variational Problem Arising from Studies of Nonlinear Charge Screening in Graphene Monolayers

17:00 - 17:20 **Lorenzo Tentarelli** – Nonlinear point interactions for the Schrödinger equation in $d = 2$

17:30 - 17:50 **Simone Dovetta** – NLS ground states on the two-dimensional grid: dimensional crossover and a continuum of critical exponents

18:00 - 18:20 **Raffaele Scandone** – Non-linear Schrödinger equation with singular potentials: new results and open problems

Saturday 24/02/2016

09:00 - 09:50 **Stefan Teufel** – Adiabatic theorems in quantum mechanics V

10:00 - 10:50 **Stefan Teufel** – Adiabatic theorems in quantum mechanics VI

11:00 - 11:30 Coffee Break

11:30 - 12:20 **Benjamin Schlein** – Bogoliubov theory for excitation spectra of interacting Bose gases V

12:30 - 13:20 **Benjamin Schlein** – Bogoliubov theory for excitation spectra of interacting Bose gases VI

13:20 - 13:30 Closing by **Gianfausto Dell'Antonio**

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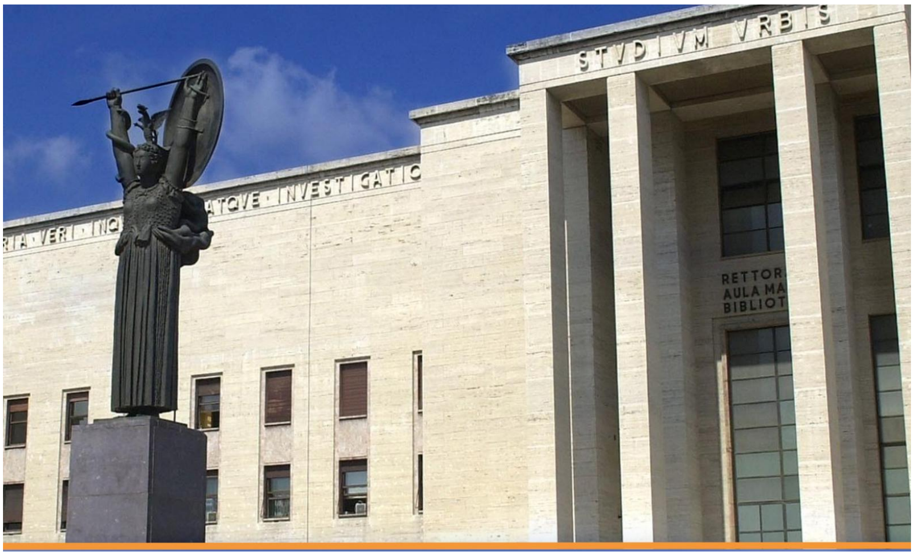
ABSTRACTS



Courses

The ionization problem

Rupert L. Frank
(Ludwig Maximilian Universität München)

The ionization conjecture says that a nucleus of charge Z can bind at most $Z+C$ electrons, where C is a constant independent of Z. (In nature, probably $C = 1$ or $C = 2$ will do.) This conjecture is still open. We review some results related to this problem and, in particular, Solovej's solution of the analogous problem in Hartree-Fock theory. We also discuss some recent contributions obtained jointly with Nam and van den Bosch.

Bogoliubov theory for excitation spectra of interacting Bose gases

Benjamin Schlein
(Universität Zürich)

We consider systems of N interacting bosons in the Gross-Pitaevskii limit, where the scattering length of the potential is of the order $1/N$ and N tends to infinity. For non-negative and sufficiently weak interactions, we establish the low-energy spectrum, i.e. the ground state energy and low-lying excitations, up to errors that vanishes in the limit of large N. As a result, we rigorously confirm the validity of Bogoliubovs 1947 predictions.

This talk is based on joint works with C. Boccato, C. Brennecke and S. Cenatiempo.

Adiabatic theorems in quantum mechanics

Stefan Teufel
(Universität Tübingen)

Starting from Kato's classical result I will discuss various developments in and applications of adiabatic perturbation theory in quantum mechanics. These developments include adiabatic theory to all orders, adiabatic theorems without spectral gap, adiabatic theorems for non-self-adjoint generators, and very recent adiabatic theorems for extended systems. Applications will be primarily from the area of solid state physics and particular emphasis will be given to geometric aspects like the role of the Berry connection.

Plenary Talks

Quantum fields and point interactions

Sergio Albeverio
(Universität Bonn)

Local nodal surplus and nodal count distribution of graphs with disjoint loops

Gregory Berkolaiko
(Texas A&M University)

We start by reviewing the notion of “quantum graph”, its eigenfunctions and the problem of counting the number of their zeros. The nodal surplus of the n-th eigenfunction is defined as the number of its zeros minus $(n-1)$. When the graph is composed of two or more blocks separated by bridges, we propose a way to define a “local nodal surplus” of a given block. Since the eigenfunction index n has no local meaning, the local nodal surplus has to be defined in an indirect way via the nodal-magnetic theorem of Berkolaiko, Colin de Verdiere and Weyand.

We will discuss the properties of the local nodal surplus and their consequences. In particular, its symmetry properties allow us to prove the long-standing conjecture that the nodal surplus distribution for graphs with β disjoint loops is binomial with parameters $(\beta, 1/2)$.

Concentration of quantum states from quantum functional and transportation cost inequalities

Nilanjana Datta
(University of Cambridge)

Quantum functional inequalities (e.g. the logarithmic Sobolev and Poincaré inequalities) have found widespread application in the study of the behavior of primitive quantum Markov semigroups. The classical counterparts of these inequalities are related to each other via a so-called transportation cost inequality of order 2 (T2). The latter inequality relies on the notion of a metric on the set of probability distributions called the Wasserstein distance of order 2. (T2) in turn implies a transportation cost inequality of order 1 (T1). In this paper, we introduce quantum generalizations of the inequalities (T1) and (T2), making use of appropriate quantum versions of the Wasserstein distances, one recently defined by Carlen and Maas and the other defined by us. We establish that these inequalities are related to each other, and to the quantum modified logarithmic Sobolev and Poincaré inequalities, as in the classical case. We also show that these inequalities imply certain concentration-type results for the invariant state of the underlying semigroup. We consider the example of the generalized depolarizing semigroup to derive concentration inequalities for any finite dimensional full-rank quantum state. These inequalities are then applied to derive upper bounds on the error probabilities occurring in the setting of finite blocklength quantum parameter estimation.

This is joint work with Cambyse Rouzé.

Universality of the Hall conductivity in interacting electron systems

Alessandro Giuliani
(Roma Tre University)

The Haldane model is a paradigmatic 2d lattice model exhibiting the integer quantum Hall effect. We prove that for short-range interactions the Hall conductivity is quantized, for all the values of the parameters outside two critical curves, across which the model undergoes a 'topological' phase transition: the Hall coefficientremains integer and constant as long as we continuously deform the parameters without crossing the curves; when this happens, the Hall coefficient jumps abruptly to a different integer. In contrast to previous work, we do not assume the interaction to be small with respect to the bare gap. The non-renormalization of the Hall conductivity arises as a simple consequence of lattice conservation laws and of the regularity properties of the current-current correlations. Our method provides a full construction of the critical curves, which are modified ('dressed') by the electron-electron interaction. The shift of the transition curves manifests itself via apparent infrared divergences in the naive perturbative series, which we resolve by exact renormalization group methods.

On the 2d Fermi-polaron

Marcel Griesemer
(Universität Stuttgart)

The Fermi polaron denotes a system composed of an ideal gas of N fermions interacting with an impurity particle. The interaction between impurity and fermions is given in terms of two-body point interactions whose strength is determined by the two-body binding energy. The self-adjoint realization requires a UV regularization and the Hamiltonian emerges in the strong resolvent limit as the UV cutoff is removed. An explicit expression is not available for the Hamiltonian but for a Birman-Schwinger type operator also known as charge operator in this context. A novel variational principle relating the spectra of the Hamiltonian and the Birman-Schwinger operator opens the door to the spectral analysis of the Fermi polaron. Going through this door we prove stability of the 2d fermi polaron as well as a conjecture in the physical literature concerning the form of the ground state in the case of strong coupling.

Bott periodicity and the "periodic table" of topological insulators and superconductors

Martin Zirnbauer
(Universität Köln)

Bott periodicity is said to be one of the most surprising phenomena in topology. Perhaps even more surprising is its recent appearance in condensed matter physics. Building on work of Schnyder et al., Kitaev argued that symmetry-protected ground states of gapped free-fermion systems, also known as topological insulators and superconductors, organize into a kind of periodic table governed by a variant of the Bott periodicity theorem. In this talk, I will sketch the mathematical background, the physical context, and some new results of this ongoing story of mathematical physics.

Contributed Talks

Universal edge transport in interacting Hall systems

Giovanni Antinucci
(Universität Zürich)

We study the edge transport properties of 2d interacting Hall systems, displaying single-mode chiral edge currents. For this class of many-body lattice models, including for instance the interacting Haldane model, we prove the quantization of the edge charge conductance and the bulk-edge correspondence. Instead, the edge Drude weight and the edge susceptibility are interaction-dependent; nevertheless, they satisfy exact universal scaling relations, in agreement with the chiral Luttinger liquid theory. Moreover, charge and spin excitations differ in their velocities, giving rise to the spin-charge separation phenomenon. The analysis is based on exact renormalization group methods, and on a combination of lattice and emergent Ward identities. The non-renormalization of the emergent chiral anomaly plays a crucial role in the proof.

Joint work with Vieri Mastropietro and Marcello Porta.

Universal low-energy behavior in a quantum Lorentz gas with Gross-Pitaevskii potentials

Giulia Basti
(Universität Zürich)

We consider a quantum particle interacting with N obstacles, independently distributed according to a common probability density, through a Gross-Pitaevskii potential. This Lorentz gas model can be considered as a simplified model for scattering of slow neutrons from condensed matter. We show convergence of the N dependent one-particle Hamiltonian to a limiting Hamiltonian depending only on the scattering length of the unscaled potential and the density of the obstacles. In this sense our model exhibits a universal behavior for N large.

Joint work with S. Cenatiempo and A. Teta.

Hamiltonian of contact interaction of Ter-Martyrosyan-Skornyakov (TMS) type

Simon Becker
(University of Cambridge)

We consider the 2+1 fermionic trimer system consisting of two identical fermions, plus a third particle of different species with zero range interaction and study the spectral properties of the Schrdinger operator. We locate the essential spectrum and provide results on the existence, monotonicity, and location of the finite discrete spectrum below the essential spectrum.

This is joint work with Alessandro Michelangeli and Andrea Ottolini.

Optical solitons in nematic liquid crystals: model with saturation effects

Juan Pablo Borgna
(Universidad de San Martin)

We present a 2-D system that couples a Schrödinger evolution equation to a nonlinear elliptic equation that models the propagation of a laser beam in a nematic liquid crystal.

$$i\partial_z u + \tfrac{1}{2}\nabla^2 u + \sin(2\theta)u = 0 \tag{1a}$$

$$\nu\nabla^2\theta - q\sin(2\theta) = -2|u|^2\cos(2\theta) \tag{1b}$$

This system arises in experimental devices designed by G. Assanto and collaborators (see [1], [2]). The nonlinear elliptic equation (1b) describes the response of the director angle to the laser beam electric field. Well-studied previous results deal with a simpler system with a linear elliptic equation for the director field, obtained from considering $\sin(2\theta) \approx 2\theta$ and $\cos(2\theta) \approx 1$; (see [3]) then system (1) becomes into

$$i\partial_z u + \frac{1}{2}\nabla^2 u + 2\theta u = 0 \tag{2a}$$

$$\nu\nabla^2\theta - 2q\theta = -2|u|^2 \tag{2b}$$

The linear elliptic equation for the director field (2b) has a solution $\theta = G * |u|^2$; where $G(x) = 2\nu^{-1}N_0(\sqrt{2q/\nu}|x|)$ and N_0 is the modify Bessel function. Thus, system (2) is reduced to the following Hartree type NLS equation:

$$i\partial_z u + \frac{1}{2}\nabla^2 u + 2\left(G * |u|^2\right)u = 0 \tag{3}$$

From equation (3) the authors were able to explain the physical effect that the presence of a well-located field u produces a deformation in the long-distance director angle and it avoids the occurrence of the expected blow up in a cubic Schrödinger equation in $\mathbb{R}^2$ (see [3]). But, since equation (3) it was not possible to explain the experimentally observed saturation effect on the director angle: θ can not be aligned with the preconditioning electric field. In this work we present an analysis of the nonlinear elliptic system (1), by using of nonlinear techniques. We show the existence of the local and global solutions of the system (1) and that the deviation θ of the director field, remains bounded in $[0, \pi/4)$ (saturation effect). At the end, for sufficiently large L^2-norm for incident laser u; we show the existence of energy minimizing optical solitons with radial, positive and monotone profiles.

Joint work with P. Panoyotaros, D Rial and C. Sanchez de la Vega.

[1] M. Peccianti y G. Assanto, *Nematicons*, Physics Reports 516(4):147-208, 2012.

[2] G. Assanto, Nematicons. *Spatial Optical Solitons in Nematic Liquid Crystals*, Wiley and Sons, 2013.

[3] J.M. McNeil, N. Smyth and G. Assanto, *Exact and approximate solutions for optical solitary waves in nematic liquid crystals*, Physics D, 284 (2014), pp.1-15.

Effective Dirac equations in honeycomb structures

William Borrelli

(Université Paris-Dauphine)

Recently, new two-dimensional materials possessing Dirac fermions low-energy excitations have been discovered, the most famous being graphene (see, e.g. [8]). In those materials electrons at the Fermi level have zero apparent mass and can be described using the massless Dirac equation. More generally, Schrödinger operators with honeycomb potentials generically exhibit conical intersections (the so-called Dirac points) in their dispersion bands [6]. This leads to the appearance of Dirac as the effective operator, describing the electron dynamics (in certain regimes). The large, but finite, time-scale validity of the Dirac approximation has been proved in [7] for the linear case, and in [1] for cubic nonlinearities. The latter case corresponds to the *Gross-Pitaevski equation*, which is a fundamental model in the description of macroscopic quantum phenomena and in nonlinear optics. The cubic Dirac equation in 2D is *critical* for the Sobolev embedding, and this makes the existence of stationary solutions a non-trivial problem [4]. Describing finite size samples of graphene requires to choose suitable boundary conditions for the Dirac operator [2]. Local well-posedness for a model of electron transport in graphene has been proved in [5], while the existence of stationary solutions has been addressed in [3].

In this talk I will give an overview of this results.

[1] J. Arbunich and C. Sparber, *Rigorous derivation of nonlinear Dirac equations for wave propagation in honeycomb structures*, ArXiv e-prints, (2016).

[2] R. D. Benguria, S. r. Fournais, E. Stockmeyer, and H. Van Den Bosch, *Self-adjointness of two-dimensional Dirac operators on domains*, Ann. Henri Poincaré, 18 (2017), pp. 1371–1383.

[3] W. Borrelli, *Multiple solutions for a self-consistent Dirac equation in two dimensions*, ArXiv e-prints, (2017).

[4] W. Borrelli, *Stationary solutions for the 2D critical Dirac equation with Kerr nonlinearity*, J. Differential Equations, 263 (2017), pp. 7941–7964.

[5] R. El Hajj and F. Méhats, *Analysis of models for quantum transport of electrons in graphene layers*, Math. Models Methods Appl. Sci., 24 (2014), pp. 2287–2310.

[6] C. L. Fefferman and M. I. Weinstein, *Honeycomb lattice potentials and Dirac points*, J. Amer. Math. Soc., 25 (2012), pp. 1169–1220.

[7] C. L. Fefferman and M. I. Weinstein, *Wave packets in honeycomb structures and two-dimensional Dirac equations*, Comm. Math. Phys., 326 (2014), pp. 251–286.

[8] T. Wehling, A. Black-Schaffer, and A. Balatsky, *Dirac materials*, Advances in Physics, 63 (2014), pp. 1–76.

Derivation of the 1d Gross-Pitaevskii equation from the 3d quantum many-body dynamics of strongly confined bosons

Lea Boßmann

(Universität Tübingen)

We consider the dynamics of N interacting bosons initially exhibiting Bose-Einstein condensation. Due to an external trapping potential, the bosons are strongly confined in two spatial directions, with the transverse extension of the trap being of order ε. The interaction potential is scaled such that its scattering length is positive and of order $(N/\varepsilon^2)^{-1}$, the range of the interaction scales as $(N/\varepsilon^2)^{-\beta}$ for $\beta \in (0,1]$. We prove that when taking simultaneously the limits $N \to \infty$ and $\varepsilon \to 0$, the condensation is preserved by the dynamics and the time evolution is asymptotically described by a nonlinear Schrödinger equation in one dimension. For $\beta = 1$, the effective equation is of Gross-Pitaevskii type, with the strength of the nonlinearity given by the scattering length of the unscaled interaction, multiplied with a factor depending on the ground state in the confined directions.

Gross-Pitaevskii Dynamics for Bose-Einstein Condensates

Christian Brennecke

(Universität Zürich)

We study the time-evolution of initially trapped Bose-Einstein condensates in the Gross-Pitaevskii regime. Under a physically motivated assumption on the energy of the initial data, we show that condensation is preserved by the many-body evolution and that the dynamics of the condensate wave function can be described by the time-dependent Gross-Pitaevskii equation. With respect to previous works, we provide optimal bounds on the rate of condensation (i.e. on the number of excitations of the Bose-Einstein condensate).

This is joint work with B. Schlein.

On the quantization of peculiar Hamiltonians

Francesco Calogero

("Sapienza" Università di Roma & INFN)

I plan to discuss some findings about the quantization of Hamiltonians featuring some peculiarities. (i) It is generally expected that Hamiltonians yielding isochronous motions i. e., motions such that all solutions are completely periodic with a fixed period shall feature, after appropriate quantization, an equispaced spectrum. But what if the Hamiltonian yields solutions which are isochronous in configuration space but not in momentum space? Such an example is discussed in a joint paper with Franois Leyvraz [1]. (ii) And what about the quantization of Hamiltonians yielding motions with friction? There indeed exist contrary to a widespread view such (time-independent) Hamiltonians, including some quite physical ones, for instance one describing the motion of a charged particle moving against friction in a plane under the influence of a constant orthogonal magnetic field ("cyclotron"). Work on this second topic is now (05.12.2017) in progress in collaboration with Franois Leyvraz.

[1] F. Calogero and F. Leyvraz, "Examples of Hamiltonians isochronous in conguration space only, and their quantization", J. Math. Phys. (submitted to, 25.10.2017).

Self-adjointness for the Dirac operator with Coulomb-type potentials

Biagio Cassano
(Basque Center for Applied Mathematics)

We investigate the self-adjointess of the Dirac operator

$$H := -i\alpha \cdot \nabla + m\beta + \mathbb{V}(x),$$

for $m \in \mathbb{R}$, and $\mathbb{V}(x) \sim \nu/|x|$: although such property is of primary interest, the phenomena are not fully understood, expecially in the case $\nu \geqslant 1$. In [1], in collaboration with Fabio Pizzichillo (BCAM), we treat the case of potentials of the type

$$\mathbb{V}(x) = \frac{1}{|x|}\left(\nu\mathbb{I}_4 + \mu\beta - i\lambda\alpha \cdot \frac{x}{|x|}\beta\right),$$

for $\nu, \mu\lambda \in \mathbb{R}$. We describe the self-adjoint realizations of the operator H in terms of the behaviour in the origin of the functions of the domain: main strategy of the proof is the study of the properties of H on the partial wave subspaces, exploiting appropriate trace inequalities and Hardy inequalities. Finally, we give a description of the distinguished extension in the critical and sub-critical case.

[1] Biagio Cassano, Fabio Pizzichillo, *Self-Adjoint Extensions for the Dirac Operator with Coulomb-Type Spherical Symmetric Potentials*, preprint, arXiv:1710.08200.

On the Bogoliubov-de Gennes Equations

Li Chen
(University of Toronto)

The Bogoliubov de Gennes (BdG) equations form a microscopic description of superconductivity. They present an equivalent approach as to the BCS theory. When the temperature T is lower than a certain critical T_c, superconducting solutions emerges. Macroscopically, when T is near T_c, the superconductor is described by the Ginzburg-Landau theory. In this talk, I will present a sketch of the derivation of the Ginzburg-Landau equations as an effective equation of the BdG equations under suitable scaling limit. The main idea of the derivation is to perform bifurcation at the normal (non superconducting) state and to employ iterated Lyapunov Schmidt maps. This technique requires rigorous examination of the linear operator (linearized at the normal state) and the nonlinear terms. For the former, I will present its spectrum analysis near the ground state while I will attempt to give optimal nonlinear estimates for the latter.

This is a joint work with I. M. Sigal.

Stability of solitary waves in nonlinear Dirac equation with concentrated nonlinearity with broken $SU(1,1)$ symmetry

Andrew Comech
(Texas A&M University)

We consider the 1D NLD with Soler-type nonlinearity concentrated at a point. We show that if the nonlinearity is perturbed so that both the Bogoliubov $SU(1,1)$-symmetry and the spatial parity are broken, then the solitary waves become linearly unstable.

This is a joint research with N. Boussaid, C. Cacciapuoti, R. Carlone, D. Noja, and A. Posilicano.

Free fermions and the classical compact groups

Fabio Deelan Cunden
(University College Dublin)

There is a close connection between the ground state of non-interacting free fermions in a box with classical (absorbing, reflecting, and periodic) boundary conditions and the eigenvalue statistics of the classical compact groups. The associated determinantal point processes can be extended in two natural directions: i) we consider the full family of admissible quantum boundary conditions (i.e., self-adjoint extensions) for the Laplacian on a bounded interval, and the corresponding projection correlation kernels; ii) we construct the grand canonical extensions at finite temperature of the projection kernels, interpolating from Poisson to random matrix eigenvalue statistics. The scaling limits in the bulk and at the edges are studied in a unified framework, and the question of universality is addressed. Whether the finite temperature determinantal processes correspond to the eigenvalue statistics of some matrix models is, a priori, not obvious. We complete the picture by constructing a finite temperature extension of the Haar measure on the classical compact groups. The eigenvalue statistics of the resulting grand canonical matrix models (of random size) corresponds exactly to the grand canonical measure of non-interacting free fermions with classical boundary conditions.

Non-existence of ground states in the translation invariant Nelson model

Thomas Norman Dam
(Aarhus Universitet)

The Nelson model is a model from quantum field theory used to describe a spinless massive particle interacting with a scalar quantum field. If there is no external potential, the model may be written as a direct integral of operators on Fock space of the form

$$H(\xi) = \Omega(\xi - d\Gamma(k)) + d\Gamma(\omega) + g\phi(v).$$

We give criteria for non-existence of ground states for $H(\xi)$. In particular if one uses the physical choices of v, ω and Ω in dimension 3, then one can conclude that $H(\xi)$ has no ground state for any choice of ξ or $g \neq 0$.

Linear Response Theory: An Analytic-Algebraic Approach

Giuseppe De Nittis
(Pontificia Universidad Católica de Chile)

Linear response theory (LRT) is a tool with which one can study the response of systems that are driven out of equilibrium by external perturbations. In this talk I present a modern and systematic approach to LRT by combining analytic and algebraic ideas. The theory is robust and provides a tool to implement LRT for a wide array of systems like periodic and random systems in the discrete and the continuum. The mathematical framework of the theory is outlined firstly: the relevant von Neumann algebras, non-commutative L^p- and Sobolev spaces are introduced; the notion of isospectral perturbations and the associated dynamics and commutators are studied; their construction is then made explicit for various physical systems (quantum systems, classical waves). The final part is dedicated to a presentation of the proofs of the Kubo and Kubo-Streda formulas.

Joint work with M. Lein.

Bose-Einstein Condensation in a Dilute, Trapped Gas at Positive Temperature

Andreas Deuchert
(IST Austria)

We consider an interacting, dilute Bose gas trapped in a harmonic potential at a positive temperature. The system is analyzed in a combination of a thermodynamic and a Gross-Pitaevskii (GP) limit where the trap frequency ω, the temperature T and the particle number

$N \to \infty$ are related by $N \sim (T/\omega)^3$ while the scattering length is so small that the interaction energy per particle around the center of the trap is of the same order of magnitude as the spectral gap in the trap. We prove that the difference between the canonical free energy of the interacting gas and the one of the non-interacting system can be obtained by minimizing the GP energy functional. We also prove Bose-Einstein condensation in the following sense: The one-particle density matrix of any approximate minimizer of the canonical free energy functional is to leading order given by the one of the non-interacting gas but with the free condensate wavefunction replaced by the GP minimizer.

Joint work with R. Seiringer and J. Yngvason.

Phase transitions for a rotating Bose-Einstein condensate: the third critical speed

Daniele Dimonte
(SISSA)

We will talk about phase transitions for a fast rotating Bose-Einstein condensate. When rotating the condensate undergoes three phase transitions. In particular the third one occurs when the rotating speed is of order $1/(\varepsilon^4)$ with respect to ε, the coupling constant; at that speed the condensate is essentially supported in a ring with vortices in it while when the speed becomes higher the vortices disappear and the condensate arranges itself as in a "giant vortex state". We prove an estimate (upper bound) of the third critical speed and discuss the possible optimality of such a value.

This talk is based on a joint work with Michele Correggi.

NLS ground states on the two-dimensional grid: dimensional crossover and a continuum of critical exponents

Simone Dovetta
(Politecnico di Torino)

The purpose of this talk is to analyse the existence of ground states for the nonlinear Schr'odinger (NLS) energy functional

$$E(u, G) = \frac{1}{2}\int_{\mathcal{G}_\ell} |u'|^2 dx - \frac{1}{p}\int_{\mathcal{G}_\ell} |u|^p dx \tag{1}$$

on the two-dimensional grid of edge-length $\ell > 0$, $\mathcal{G}_\ell$, under the mass constraint

$$\int_{\mathcal{G}_\ell} |u|^2 dx = \mu.$$

We deal with this problem both in the subcritical and in the critical regime, i.e. when $p \in (2, 6)$ and $p = 6$ respectively. We show that, when $p \in (2, 4)$, ground states of (1) exist for every value of the mass. On the contrary, for every $p \in (4, 6]$, a phase transition occurs and ground states exists if and only if the mass is greater or equal than a threshold value (depending on the exponent). Finally, if $p = 4$, we prove that ground states exist when μ is greater than a critical value and they do not exist when μ is smaller. We explain how these phase transitions are rooted in the coexistence of purely one-dimensional and two-dimensional behaviour on the grid, showing that this situation allows to derive for $\mathcal{G}_\ell$ a whole new family of Gagliardo-Nirenberg inequalities, interpolating between the one-dimensional and the two- dimensional versions.

Joint work with R. Adami, E. Serra and P. Tilli.

Global exact controllability of the bilinear Schrödinger potential type models on compact quantum graphs

Alessandro Duca
(Politecnico di Torino & Université de Bourgogne Franche-Comté)

In quantum mechanics any pure state of a system is mathematically represented by a wave function in the unit sphere of a Hilbert space $\mathscr{H}$. The dynamics of a particle constrained in a compact graph structure $\mathscr{G}$ and excited by a controlled field is represented by the Cauchy problem in $\mathscr{H} = L^2(\mathscr{G}, \mathbb{C})$

$$\begin{cases} i\partial_t \psi(t) = A\psi(t) + u(t)B\psi(t), \\ \psi(0) = \psi^0 \qquad t \in (0, T). \end{cases} \tag{1}$$

The operator B is bounded symmetric, u is a control function and ψ^0 is the initial state of the system. The operator $A = -\Delta$ is the Laplacian equipped with self-adjoint type boundary conditions into the vertices of the graph. We present the work [1] where we study the problem (1) for $\mathscr{G}$ a generic compact quantum graph. In particular, we analyze how the boundary conditions and the structure of the graph affect its controllability. Provided the well-posedness of (1), we present assumptions on B and on the spectrum of A implying the global exact controllability in suitable subspaces of $\mathscr{H}$. When the previous assumptions fail, we introduce a weaker notion of controllability. In particular, we define the so-called "energetic controllability" which allows to provide interesting results also when $\mathscr{G}$ is a complex structure and we are not able to verify the spectral assumptions for the global exact controllability.

[1] Alessandro Duca. Global exact controllability of the bilinear Schrödinger potential type models on quantum graphs. *http://arxiv.org/abs/1710.06022*.

Friedrichs operators as dual pairs and contact interactions

Marko Erceg
(University of Zagreb)

The Friedrichs (1958) theory of positive symmetric systems of first order partial differential equations encompasses many standard equations of mathematical physics, irrespective of their type. This theory was recast in an abstract Hilbert space setting by Ern, Guermond and Caplain (2007), and by Antonić and Burazin (2010). In this work we make a further step, presenting a purely operator-theoretic description of abstract Friedrichs systems via the universal operator extension theory of dual pairs (Grubb, 1968). For a given Friedrichs system the existence of a boundary condition such that the problem is well-posed is shown, as well as a classification of all such boundary conditions. Within this framework we study self-adjoint (or even only closed) realisations of the Hamiltonians of contact interactions.

Joint work with Nenad Antonić and Alessandro Michelangeli.

On the spectrum of the two-dimensional Schrödinger Hamiltonian with the isotropic harmonic confinement or the isotropic pyramidal one in the presence of a central point perturbation

Silvestro Fassari
(Universidad de Valladolid)

In this brief presentation the discrete spectra of the Hamiltonian of the two-dimensional isotropic harmonic oscillator perturbed by a point interaction centred at the origin and that of the two-dimensional Hamiltonian in which the isotropic harmonic confinement is replaced by the isotropic pyramidal one will be compared. Level crossings of eigenvalues occurring in both models will be analysed in detail.

Joint work with M. Gadella, M.L. Glasser, L.M. Nieto and F. Rinaldi.

Some results on scattering theory for delta interactions concentrated on deformed planes

Davide Fermi
(Università degli Studi di Milano)

We consider singular perturbations of the free Laplacian corresponding to surface delta interactions concentrated on a flat plane π_0 in $\mathbb{R}^d$ or on a deformation π_F of it, determined by a compactly supported shape function F. We construct the corresponding resolvent operators $R_{\pi_0}(z)$, $R_{\pi_F}(z)$ by means of standard Krein-like formulas, where the reference operator is the Laplacian on $H^2(\mathbb{R}^d)$. Next, we use the expressions thus obtained to study the scattering problem for the two dynamics associated to π_0 and π_F, respectively. More precisely, we prove that the associated wave operators exists and are complete for $d \leqslant 3$, which follows from the Birman-Kato criterion noting that the resolvent difference $R_{\pi_0}(z) - R_{\pi_F}(z)$ is trace class. In view of future developments regarding the scattering matrix, we also provide limiting absorption principles for $R_{\pi_0}(z)$ and $R_{\pi_F}(z)$ holding in suitable weighted L^2 spaces; this is achieved employing previous results of Ben-Artzi, Devinatz and Renger.

Joint work with Claudio Cacciapuoti and Andrea Posilicano.

Feynman graphs and the large dimensional limit of multipartite entanglement

Giuseppe Florio
(Politecnico di Bari)

We extend the analysis of multipartite entanglement based on techniques from classical statistical mechanics [1-3] to a system composed of n d-level parties (qudits). We consider the purity of a subsystem as a measure of bipartite entanglement and its average among balanced bipartitions as a measure of multipartite entanglement. We introduce a suitable partition function at a fictitious temperature with the average local purity of the system as Hamiltonian. In particular, we analyze the high-temperature expansion of this partition function, prove the convergence of the series and study its asymptotic behavior as d goes to infinite. We make use of a diagrammatic technique, classify the graphs and study their degeneracy. We are thus able to evaluate their contributions and estimate the moments of the distribution of the local purity [4].

[1] P. Facchi, G. Florio, U. Marzolino, G. Parisi, and S. Pascazio, "Statistical mechanics of multipartite entanglement", J. Phys. A: Math. and Theor. 42, 055304 (2009).

[2] P. Facchi, G. Florio, U. Marzolino, G. Parisi, and S. Pascazio, "Classical statistical mechanics approach to multipartite entanglement", J. Phys. A: Math. and Theor. 43, 225303 (2010).

[3] P. Facchi, G. Florio, U. Marzolino, G. Parisi, and S. Pascazio, "Multipartite entanglement and frustration", New Jour. Phys. 12, 025015 (2010).

[4] S. Di Martino, P. Facchi, G. Florio, "Feynman graphs and the large dimensional limit of multipartite entanglement", J. Math. Phys., in press (2018).

Self-adjoint realisations of the Dirac-Coulomb Operator

Matteo Gallone
(SISSA)

Central to relativistic quantum mechanics is the Dirac equation, a partial differential equation that describes the dynamics of a spin-$1/2$ particle. When one studies such dynamics in the electrostatic field generated by a nuclear point charge placed at the origin, one has to face the problem of qualifying the self-adjoint realisation(s) of the minimal Dirac-Coulomb operator. In this talk I will present some results concerning the classification of self-adjoint realisations and their spectra obtained in collaboration with Alessandro Michelangeli.

On evolution equations of processes of creation and propagation of correlations in quantum systems

Viktor Gerasimenko

(Institute of Mathematics of the National Academy of Sciences of Ukraine)

We consider the problem of the rigorous description of the evolution of states of large particle quantum systems by means of marginal correlation operators. The physical interpretation of marginal correlation operators is that the macroscopic characteristics of fluctuations of mean values of observables are determined by them on the microscopic level. In particular, considered problem is related to the problem of an entanglement of quantum states. As a result of the definition of the marginal correlation operators within the framework of dynamics of correlations governed by the von Neumann hierarchy [1] we establish that a sequence of such operators is governed by the nonlinear quantum BBGKY (Bogolyubov–Born–Green–Kirkwood–Yvon) hierarchy and a nonperturbative solution of the Cauchy problem to this hierarchy of nonlinear evolution equations is represented in the form of series expansions over the number of particles of subsystems which generating operators are the corresponding-order cumulants of the groups of nonlinear operators of the von Neumann hierarchy for a sequence of correlation operators. Moreover, the concept of quantum kinetic equations in case of initial states specified in terms of a one-particle density operator and correlation operators, for instance, the initial correlation operators, characterizing the condensed states or their influence on ultrafast relaxation processes in plasmas, was considered. In particular, we established that a mean field behavior of processes of the creation of correlations and the propagation of initial correlations in large particle quantum systems are governed by the Vlasov-type quantum kinetic equation with initial correlations [2]. In case of pure states derived kinetic equation can be reduced to the Gross-Pitaevskii kinetic equation or to the nonlinear Schrödinger equation.

[1] V. I. Gerasimenko, "Hierarchies of quantum evolution equations and dynamics of many-particle correlations", In: *Statistical Mechanics and Random Walks: Principles, Processes and Applications*, N.Y.: Nova Science Publ., Inc., 2013, 233–288.

[2] V. I. Gerasimenko, "Processes of creation and propagation of correlations in quantum many-particle systems", *Reports of NAS of Ukraine*, No. 5, 58–66 (2016).

An adiabatic approach to constrained quantum systems with spin-orbit interaction

Matthias Geyer

(Technische Universität Dresden)

In order to explain spin polarization of electrons traveling through chiral molecules [1, 2], we investigate a one-particle Pauli-Hamiltonian with helical confinement and confinement-induced spin-orbit coupling (SOC) as proposed in [3, 4]. Aiming for a one-dimensional approximation of this model, we explore the adiabatic decoupling behavior of confined quantum systems with SOC applying methods from [5, 6]. We show that in the presence of SOC, the approximation error will only be small if the adiabatic scaling parameter takes values within an interval with small upper and positive lower bound.

[1] K Ray, S. P. Ananthavel, David H. Waldeck, and Ron Naaman. Asymmetric Scattering of Polarized Electrons by Organized Organic Films of Chiral Molecules. *Science*, 283(5403):814–816, 1999.

[2] Ron Naaman and David H Waldeck. Spintronics and Chirality: Spin Selectivity in Electron Transport Through Chiral Molecules. *Annual review of physical chemistry*, 66(January):263–281, 2015.

[3] Karen Michaeli, Vaibhav Varade, Ron Naaman, and David H Waldeck. A New Approach towards Spintronics - Spintronics with No Magnets. *Journal of Physics: Condensed Matter*, 29(10):103002, 2017.

[4] Karen Michaeli and Ron Naaman. Origin of Spin Dependent Tunneling through Chiral Molecules. 2015.

[5] Herbert Spohn and Stefan Teufel. Adiabatic decoupling and time-dependent Born-Oppenheimer theory. *Communications in Mathematical Physics*, 224(1):113–132, 2001.

[6] Stefan Teufel. *Adiabatic Perturbation Theory in Quantum Dynamics*. Springer Science & Business Media, 2003.

Surface Superconductivity in Presence of Corners

Emanuela L. Giacomelli
(Universität Tübingen)

We consider an extreme type-II superconducting wire with non-smooth cross section, i.e. with one or more corners at the boundary, in the framework of the Ginzburg-Landau theory. We prove the existence of an interval of values of the applied field, where superconductivity is spread uniformly along the boundary of the sample. The critical fields delimiting this surface superconductivity regime coincide with the ones in absence of boundary singularities. The energy is not affected to leading order by the presence of corners. To isolate the contributions to the energy density due to the presence of corners we then introduce a new effective problem. We still do not have an explicit expression of the effective energy, but we formulate a conjecture on it based on the behaviours for almost flat angles.

Joint work with Michele Correggi.

Derivation of the Hartree equation for compound Bose gases in the mean field limit

Michael Hott
(University of Texas)

In 1925, Bose and Einstein postulated the condensation of Bosons at low temperatures. It took 70 years, to experimentally verify this condensation. However, mathematicians have been trying to rigorously establish this condensation in the formulation of quantum statistics. The derivation of the Hartree equation from many-body systems of Bosons in the mean field limit has been very intensively studied in the last couple of years. However, very few results exist showing convergence of the k-th marginal of the N-body density matrix to the projection to the k-fold tensor product of the solution of the Hartree equation in stronger trace norms like the energy trace norm. This issue is from a physical view point very important. The reason is that one can then approximate expectation values of certain observables of the N-body system by means of the Hartree equation, with relaxation of the very restrictive assumption that the observables are bounded operators. I will give a brief introduction into the concept of BEC, discuss some recent results and then show how one can upgrade previous results by using simple interpolation.

Spectral analysis of the spin-boson Hamiltonian with two photons for arbitrary coupling

Orif Ibrogimov
(University College London)

We study the spectrum of the spin-boson model with two photons in $\mathbb{R}^d$ for arbitrary coupling $\alpha > 0$ under “minimal” smoothness and regularity conditions on the photon dispersion and the coupling function.

Nematic liquid crystal phase in a system of interacting dimers

Ian Jauslin
(Institute for Advanced Study)

In 1979, O. Heilmann and E.H. Lieb introduced an interacting dimer model with the goal of proving the emergence of a nematic liquid crystal phase in it. In such a phase, dimers spontaneously align, but there is no long range translational order. Heilmann and Lieb proved that dimers do, indeed, align, and conjectured that there is no translational order. I will discuss a recent proof of this conjecture.

This is joint work with Elliott H. Lieb.

The adiabatic behaviour of a bound state diving into continuum

Hans Konrad Knörr
(Aalborg Universitet)

We study the survival probability of a bound state when an external potential varies smoothly and adiabatically in time. The initial state corresponds to a discrete eigenvalue which dives into the continuous spectrum and re-emerges from it as the potential is varied in time and finally returns to its initial value. Our main result is that the survival probability of this bound state vanishes in the adiabatic limit. The methods used in the proof are quite robust and may be adopted to cover a large class of operators, including Schrödinger and Dirac operators.

This talk is based on joint work with H. Cornean, A. Jensen and Gh. Nenciu.

Rate of Convergence towards Hartree Dynamics with Singular Interaction Potential

Jinyeop Lee
(KAIST)

We consider a system of N-Bosons with a two-body interaction potential $V \in L^2(\mathbb{R}^3) + L^\infty(\mathbb{R}^3)$, possibly singular than the Coulomb interaction. We show that, with $H^1(\mathbb{R}^3)$ initial data, the difference between the many-body Schrödinger evolution in the mean-field regime and the corresponding Hartree dynamics is of order $1/N$, for any fixed time. The N-dependence of the bound is optimal.

Joint work with Li Chen and Ji Oon Lee.

Quantum boundary conditions

Marilena Ligabò
(Università degli Studi di Bari)

We establish a bijection between the self-adjoint extensions of the Laplace operator on a bounded regular domain and the unitary operators on the boundary. Each unitary encodes a specific relation between the boundary value of the function and its normal derivative. This bijection sets up a characterization of all physically admissible dynamics of a nonrelativistic quantum particle confined in a cavity. Moreover we consider the effect of a rapid switching between two different boundary conditions. We show that this procedure induces, in the limit of infinitely frequent switchings, a new effective dynamics in the cavity related to a novel boundary condition. We obtain a dynamical composition law for boundary conditions which gives the emerging boundary condition in terms of the two initial ones.

A generalization of the Friedrichs-Lee Hamiltonian and its singular coupling limit

Davide Lonigro
(Università degli Studi di Bari)

The Friedrichs-Lee Hamiltonian models the interaction of a nondegenerate two-level system (e.g. atom) with a bosonic bath, provided that one excitation is available to the system. We propose a generalization of the model in which the space of momenta is taken as a generic measure space $(X, \mathrm{d}\mu)$ and the dispersion relation is a measurable real-valued function $\omega : X \to \mathbb{R}$; the coupling between atom and field is controlled by a square-integrable form factor $g \in \mathscr{L}^2(X, \mathrm{d}\mu)$. We also show that, under suitable conditions, the formalism can be extended to include a form factor $g \notin \mathscr{L}^2(X, \mathrm{d}\mu)$ (singular coupling), and we show that the dynamics of the excited state is completely characterized by a Borel measure that weighs the values of energy; in particular, the insurgence of exponential decay is studied. Finally, we study a class of generalized Friedrichs-Lee Hamiltonians exhibiting a peculiar evolution of the excited state: it decay exactly exponentially up to a time T which depends on the choice of the spectral measure, thereafter oscillations take place. This is an example of hidden non-Markovianity, since non-Markovian effects are only noticeable when the physical time scale of the dynamics is less than T.

Kubo formula for the quantum (spin) Hall conductivity: a microscopic derivation

Domenico Monaco
(Università degli Studi Roma Tre)

I will discuss the derivation from microscopic principles of Kubo-like formulas for linear response coefficients of conductivities, within the one-particle approximation. A gapped periodic Hamiltonian H_0 is perturbed by adding a constant electric field of intensity $\varepsilon \ll 1$ in the j-th direction, and the response of an adiabatic current in the i-th direction, modelled by a current operator of the form$i/\varepsilon[H_0, SX_i]$, is computed to the first relevant order in ε via space-adiabatic perturbation theory. Here S is an operator acting on the internal degrees of freedom of the system only. In applications to 2-dimensional Hall systems, S could represent the charge (quantum Hall effect) or spin (quantum spin Hall effect) of the particle. For these systems, we recover the quantization of the response coefficient when S is a conserved quantity.

This is joint work with Giovanna Marcelli and Stefan Teufel.

Universal relations in non-equilibrium CFT

Per Moosavi
(KTH Royal Institute of Technology)

In equilibrium, conformal field theory (CFT) is known to describe the universal critical behavior of generic models at low temperatures. In this talk I will present universal non-equilibrium results for heat and charge transport that hold for any unitary CFT. The results include the full time evolution from non-equilibrium initial states defined by position-dependent temperature and chemical-potential profiles and the resulting non-equilibrium steady state reached in the long-time limit. In particular, a simple universal correspondence between initial profiles and wave propagation in CFT is established.

Based on *arXiv:1712.00141* together with K. Gawedzki and E. Langmann.

Groundstates and radial solutions to Schrödinger-Poisson-Slater equations at the critical frequency

Vitaly Moroz
(Swansea University)

Schrödinger-Poisson-Slater (SPS) equation is derived as a mean-filed limit of the linear N-body Schrödinger problem for fermionic particles. We discuss the Coulomb-Sobolev function spaces, which appear as the natural domain for the energy functional of a class of equations of SPS type, and establish a family of optimal interpolation inequalities associated with the Coulomb-Sobolev spaces. We also prove the existence of optimizers for the inequalities, which implies the existence of ground-states to SPS equations for a certain range of the parameters. Finally, we derive radial Strauss type estimates and use them to prove the existence of radial solutions to SPS equations in a range of parameters which is in general wider than the range of existence parameters obtained via interpolation inequalities. The latter suggests a striking radial symmetry breakup conjecture.

This is based on joint works with Jacopo Bellazzini (Sassari), Marco Ghimenti (Pisa), Carlo Mercuri (Swansea) and Jean Van Schaftingen (Louvain-la-Neuve).

Dirac operators with Coulomb potentials

Sergey Morozov
(LMU München)

We will review some recent results in the spectral theory of Dirac operators with Coulomb potentials in dimensions 1, 2 and 3. Starting from self-adjoint realisations we will pass to lower bounds on moduli of Coulomb-Dirac operators in terms of fractional Laplacians and conclude with estimates on the eigenvalues for perturbed projected models. If time permits, we will also discuss applications to different quantum systems.

Stability of a fermionic $N+1$ particle system with point interactions

Thomas Moser
(IST Austria)

Unlike the bosonic case where point interactions lead to instability because of the Thomas/Efimov effect, stability can be proven for a fermionic system under suitable conditions. In particular the $2+1$ fermionic system is well understood and it turns out that there is a critical mass ratio determining stability. Recently, we were able to show stability for the $N+1$ fermion model allowing for arbitrary many particles extending the preliminary results which were restricted to few body problems. Furthermore, I will discuss energy bounds for the confined system with a constant density.

A Non-local Variational Problem Arising from Studies of Nonlinear Charge Screening in Graphene Monolayers

Cyrill Muratov
(New Jersey Institute of Technology)

This talk is concerned with energy minimizers in an orbital-free density functional theory that models the response of massless fermions in a graphene monolayer to an out-of-plane external charge. The considered energy functional generalizes the Thomas-Fermi energy for the charge carriers in graphene layers by incorporating a von-Weizsaecker-like term that penalizes gradients of the charge density. Contrary to the conventional theory, however, the presence of the Dirac cone in the energy spectrum implies that this term should involve a fractional Sobolev norm of the square root of the charge density. We formulate a variational setting in which the proposed energy functional admits minimizers in the presence of an out-of-plane point charge. The associated Euler-Lagrange equation for the charge density is also obtained, and uniqueness, regularity and decay of the minimizers are proved under general conditions. In addition, a bifurcation from zero to non-zero response at a finite threshold value of the external charge is proved.

Many-body blow-up profile of ground states of boson stars

Dinh Thi Nguyen
(LMU München)

We study ground states of a system of N identical bosons in $\mathbb{R}^3$, described by the Hamiltonian

$$H_N = \sum_{i=1}^{N} \left(\sqrt{-\Delta_i + m^2} + V(x_i) \right) - \frac{a_N}{N-1} \sum_{1 \le i < j \le N} |x_i - x_j|^{-1}$$

acting on $\mathfrak{h}^N = \bigotimes_{\rm sym}^N L^2(\mathbb{R}^3)$. Here the parameter $m > 0$ is the mass of particles, the parameter $a_N > 0$ describes the strength of the interaction, and $V \ge 0$ is an external potential. We are interested in the behavior of the ground state energy per particle of H_N

$$e_N^Q := N^{-1} \inf_{\Psi \in \mathfrak{h}^N, \|\Psi\|_2 = 1} \langle \Psi, H_N \Psi \rangle ,$$

and the corresponding ground state when $N \to \infty$ and a_N approaches a critical number a^* (the Chandrasekhar limit) from below. We first consider the effective model: The pseudo-relativistic Hartree functional of a boson stars

$$\mathcal{E}_a^H[u] = \left\| (-\Delta + m^2)^{1/4} u \right\|_2^2 - \frac{a}{2} \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} \frac{|u(x)|^2 |u(y)|^2}{|x-y|} \mathrm{d}x \mathrm{d}y + \int_{\mathbb{R}^3} V(x) |u(x)|^2 \mathrm{d}x$$

and we study behavior of ground state energy

$$e^H(a) := \inf \left\{ \mathcal{E}_a^H[u]\, u \in H^{1/2}(\mathbb{R}^3), \|u\|_2 = 1 \right\}$$

as well as the blow-up behavior of ground states when $a \to a^*$.

Ground state properties of mixtures of condensates

Alessandro Olgiati
(SISSA)

I will present a rigorous proof of the ground state energy asymptotics for multi-component condensates. Such systems consist of multiple species of identical bosons, and their mathematical study has become topical very recently. I will show that, both in the mean field and Gross-Pitaevskii regime, the leading order of the ground state energy is captured by the minimum of a suitable one- body non-linear functional. Moreover, the ground state exhibits condensation in the sense of reduced density matrices. In the mean field regime, by an implementation of Bogoliubov theory, we are also able to compute the next-to-leading order of the ground state energy asymptotics, and to prove a norm approximation for the ground state. All our results hold under a miscibility condition, as is often called in physics literature, that allows us to prove uniqueness of the minimizer of the non-linear theory.

This is a joint work with Alessandro Michelangeli and Phan Thanh Nam.

Quasi-Classical Limit for the Pauli-Fierz Model

Marco Olivieri
("Sapienza" Università di Roma)

Pauli-Fierz model is a model describing a quantum system of interaction between a finite number of charged non-relativistic particles and an electromagnetic field. Under the assumption of high intensity of the field, quantum effects become negligible and the field can be approximated by its classical counterpart thanks to a limit procedure. Indeed, tracing out the degrees of freedom of the field, we prove that the partial trace of the full Hamiltonian converges, in resolvent sense, to an effective Schrödinger operator with magnetic field and a corrective electric potential that depends on the field configuration. Furthermore, we prove the convergence of the ground state energy of the microscopic system to the infimum over all possible classical field configurations of the ground state energy of the effective Schrödinger operator. At the end we present future perspectives about the study of the dynamics using the same approach of classical limit of the field.

Joint work with M. Correggi and M. Falconi.

Control of fuctuations and heavy tails for heat variation in the two-time measurement framework

Annalisa Panati
(Université de Toulon)

We study heat fluctuations in the two-time measurement framework in the context of quantum statistical mechanics. We show the heat fluctuation description differs considerably form it classical counterpart, in particular a crucial role is played by ultraviolet regularity conditions. For bounded perturbations, we give sufficient ultraviolet regularity conditions on the perturbation for the moments of the heat variation to be uniformly bounded in time, and for the Fourier transform of the heat variation distribution to be analytic and uniformly bounded in time in a complex neighborhood of 0. On a set of canonical examples, with bounded and unbounded perturbations, we show that our ultraviolet conditions are essentially necessary. If the form factor of the perturbation does not meet our assumptions, the heat variation distribution exhibits heavy tails. The tails can be as heavy as preventing the existence of a fourth moment of the heat variation.

Joint work with T. Benoist and R. Raquépas.

Dynamics of the nonlinear Klein-Gordon equation in the nonrelativistic limit

Stefano Pasquali
(Università degli Studi Roma Tre)

In this talk we present some results about the nonrelativistic limit ($c \to \infty$) of the nonlinear Klein-Gordon (NLKG) equation on M, where M is either $\mathbb{R}^d$ or a d-dimensional smooth compact manifold. The problem has been extensively studied over more than thirty years, and essentially all known results show convergence of solutions of the NLKG equation to solutions of the nonlinear Schrödinger (NLS) equation for times of order $O(1)$, locally uniformly in time (see [6], [2] and [3] for the case $M = \mathbb{R}^d$, and [1] for $M = \mathbb{T}^d$).
By combining techniques of Hamiltonian perturbation theory and results within the framework of dispersive PDEs, in [4] we perform r steps ($r \geq 1$) of normal form procedure in order to construct a higher-order normalized approximation of NLKG (which corresponds to the NLS at order $r = 1$), and we prove that solutions of the approximating equation approximate solutions of the NLKG equation locally uniformly in time. Furthermore, in [5] we prove that when $r \geq 2$ and $M = \mathbb{R}^d$, $d \geq 2$, small radiation solutions of the order-r normalized equation approximate solutions of the NLKG up to times of order $O(c^{2(r-1)})$.

[1] E. Faou, and K. Schratz, *Asymptotic preserving schemes for the Klein-Gordon equation in the non-relativistic limit regime*, Numerische Mathematik, 126 (3), 441–469 (2014)

[2] S. Machihara, K. Nakanishi, and T. Ozawa, *Nonrelativistic limit in the energy space for nonlinear Klein-Gordon equations*, Mathematische Annalen, 322 (3), 603–621 (2002)

[3] N. Masmoudi, and K. Nakanishi, *From nonlinear Klein-Gordon equation to a system of coupled nonlinear Schrdinger equations*, Mathematische Annalen, 324 (2), 359–389 (2002)

[4] S. Pasquali, *Dynamics of the nonlinear Klein-Gordon equation in the nonrelativistic limit, I*, preprint, arXiv:1703.01609

[5] S. Pasquali, *Dynamics of the nonlinear Klein-Gordon equation in the nonrelativistic limit, II*, preprint, arXiv:1712.03768

[6] M. Tsutsumi, *Nonrelativistic approximation of nonlinear Klein-Gordon equations in two space dimensions*, Nonlinear Analysis: Theory, Methods & Applications, 8 (6), 637–643 (1984).

From Hartree dynamics to the relativistic Vlasov equation

Simone Rademacher
(Universität Zürich)

We derive the relativistic Vlasov equation from quantum Hartree dynamics for fermions with relativistic dispersion in the mean-field scaling, which is naturally linked with an effective semiclassic limit. Similar results in the non-relativistic setting have been recently obtained by Benedikter, Porta, Saffirio and Schlein. The new challenge that we have to face here, in the relativistic setting, consists in controlling the difference between the quantum kinetic energy and the relativistic transport term appearing in the Vlasov equation.
This is joint work with Elia Dietler and Benjamin Schlein.

Non-linear Schrödinger equation with singular potentials: new results and open problems

Raffaele Scandone
(SISSA)

A central topic in mathematical physics is the rigorous investigation of many body quantum systems subject to very short range interactions. The dynamics of such systems can be efficiently described by non-linear Schrödinger equations with singular potentials. In this talk, I will discuss a recent result on the well-posedness of the Hartree equation with a point interaction in $\mathbb{R}^3$, in a suitable class of singular Sobolev spaces. I will also discuss various open problems.
Based on joint works with G. Dell'Antonio, V. Georgiev, F. Iandoli, A. Michelangeli, A. Olgiati, and K.Yajima.

Weyl quantization and lattice-gauge theory

Alexander Stottmeister
(Università degli Studi di Roma "Tor Vergata")

Weyl quantization and an adapted pseudo-differential calculus may serve as powerful tool to discuss the semi-classical limit of quantum system. We will present some results regarding the construction of a Weyl quantization for lattice-gauge theories defined via projective limits of (finite) graphs. Moreover, we will approach the problem of defining associated symbol spaces and their pseudo-differential calculus.

Conductivity and conductance: analysis of Kubo-like spin transport in topological insulators

Clément Tauber
(ETH Zürich)

In the context of quantum spin Hall effect and topological insulators, the definition of relevant observables associated to spin transport is still debated. Mostly because there are non-equivalent definitions of the spin current operator when spin is not conserved. In this talk we present our contribution to the debate by studying the transverse spin conductivity and conductance in the context of linear response theory and based on the so-called proper spin current definition. Our main theorem states that these two quantities are equal in a discrete and periodic setting, under the assumption that the spin torque linear response vanishes on average. Unlike charge transport in quantum Hall effect, this result is by no means straightforward and actually requires the definition of well suited trace-like functionals.

This is a joint work with Gianluca Panati and Giovanna Marcelli.

Nonlinear point interactions for the Schrödinger equation in $d = 2$

Lorenzo Tentarelli
("Sapienza" Università di Roma)

I will talk about the Schrödinger equation with nonlinear point interactions in dimension two. Precisely, I will start by showing that the associated Cauchy problem is locally well-posed, both in the attractive and in the repulsive case, and that mass and energy are preserved quantities along the flow. Then, I will show that in the repulsive case global existence can be guaranteed, while in the attractive case one can exhibit a class on initial data that give rise to blow-up solutions.

An homogenization approach for the inverse spectral problem of periodic Schrödinger operators

Lorenzo Zanelli
(Università di Padova)

We provide some results on the inverse spectral problem for Schrödinger operators of type $-\frac{1}{2}\hbar^2\Delta_x + V(x)$ for smooth potentials on the flat torus $\mathbb{T}^n := (\mathbb{R}/2\pi\mathbb{Z})^n$. In particular, we show how the isospectrality of two such operators is related to the periodic homogenization of the Hamilton-Jacobi equation. In the one dimensional case, we show that this connection is also related to the Bohr-Sommerfeld quantization rules.